An exact algorithm for Likelihood-based Imprecise Regression in the case of simple linear regression with interval data

Andrea Wiencierz and Marco E. G. V. Cattaneo
Department of Statistics, LMU Munich

SMPS 6, Konstanz, Germany
October 4, 2012
Likelihood-based Imprecise Regression (LIR)
Likelihood-based Imprecise Regression (LIR)

- \((X_1, Y_1), \ldots, (X_n, Y_n)\)
  with \((X_i, Y_i) \sim P\)
Likelihood-based Imprecise Regression (LIR)

- \((X_1, Y_1), \ldots, (X_n, Y_n)\) with \((X_i, Y_i) \sim P\)
- Simple linear regression: \(Y = f(X) = a + bX\)
(Simple) linear LIR with interval data

- \((X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)\)

where \(X_i^* = [\underline{X}_i, \overline{X}_i]\)

and \(Y_i^* = [\underline{Y}_i, \overline{Y}_i]\)
(Simple) linear LIR with interval data

- \((X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)\)
  where \(X_i^* = [X_i, \overline{X}_i]\)
  and \(Y_i^* = [\underline{Y}_i, \overline{Y}_i]\)

- with \(V_i^* = X_i^* \times Y_i^*\)

\(((X_i, Y_i), V_i^*)\text{ i.i.d. } P\)

such that for \(\varepsilon \in [0, 1]\)

\[ P((X_i, Y_i) \notin V_i^*) \leq \varepsilon \]
(Simple) linear LIR with interval data

- \((X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)\)
  where \(X_i^* = [\underline{X}_i, \overline{X}_i]\)
  and \(Y_i^* = [\underline{Y}_i, \overline{Y}_i]\)
- with \(V_i^* = X_i^* \times Y_i^*\)
  \(((X_i, Y_i), V_i^*) \sim P\)
  such that for \(\varepsilon \in [0, 1]\)
  \(P((X_i, Y_i) \notin V_i^*) \leq \varepsilon\)
- simple linear regression:
  \(Y = f(X) = a + bX\)
(Simple) linear LIR with interval data

- \((X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)\)
  where \(X_i^* = [X_i, \bar{X}_i]\)
  and \(Y_i^* = [Y_i, \bar{Y}_i]\)
- with \(V_i^* = X_i^* \times Y_i^*\)
  \(((X_i, Y_i), V_i^*)\) \(\text{i.i.d.} \sim P\)
  such that for \(\varepsilon \in [0, 1]\)
  \(P\left((X_i, Y_i) \notin V_i^*\right) \leq \varepsilon\)
- simple linear regression:
  \(Y = f(X) = a + b X\)
- \(p\)-quantile \(Q_{R_f,p}\), with \(p \in (0, 1)\), of the distribution of the residuals
  \(R_{f,i} = |Y_i - f(X_i)|\)
(Simple) linear LIR with interval data

- imprecise residuals:

\[
\underline{r}_{f,i} = \min_{(x,y) \in v_i^*} |y - f(x)|
\]

\[
\bar{r}_{f,i} = \sup_{(x,y) \in v_i^*} |y - f(x)|
\]
(Simple) linear LIR with interval data

- imprecise residuals:
  \[ r_{f,i} = \min_{(x,y) \in v_i^*} |y - f(x)| \]
  \[ \bar{r}_{f,i} = \sup_{(x,y) \in v_i^*} |y - f(x)| \]

- uncertainty about \( f \):
  data imprecision and statistical uncertainty
(Simple) linear LIR with interval data

- imprecise residuals:
  \[ r_{f,i} = \min_{(x,y) \in v_i^*} |y - f(x)| \]
  \[ \bar{r}_{f,i} = \sup_{(x,y) \in v_i^*} |y - f(x)| \]

- uncertainty about \( f \):
data imprecision and statistical uncertainty

- consider \( C_{f,p,\beta,\epsilon} \):
  likelihood-based confidence region for \( Q_{R_f,p} \) with cutoff point \( \beta \in (0, 1) \)
(Simple) linear LIR with interval data

- imprecise residuals:
  \[ r_{f,i} = \min_{(x,y) \in v_i^*} |y - f(x)| \]
  \[ \bar{r}_{f,i} = \sup_{(x,y) \in v_i^*} |y - f(x)| \]

- uncertainty about \( f \):
data imprecision and statistical uncertainty

- consider \( C_{f,p,\beta,\varepsilon} \):
  likelihood-based confidence region for \( Q_{R_{f,p}} \) with cutoff point \( \beta \in (0,1) \)

- result \( \mathcal{U} \): set of all plausible functions
Recapitulation: (simple) linear LIR with interval data

\[ \hat{Y}_i = f(X_i), \quad f \in F = \{ f_{a, b} : \mathbb{R} \to \mathbb{R}, X \mapsto a + b X \}, a, b \in \mathbb{R} \]
Recapitulation: (simple) linear LIR with interval data

- $((X_i, Y_i), V_i^*) \overset{i.i.d.}{\sim} P$, $P \in \mathcal{P}_\varepsilon = \{P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon\}$, $\varepsilon \in [0, 1]$
Recapitulation: (simple) linear LIR with interval data

- \(((X_i, Y_i), V_i^*)\) \(\overset{\text{i.i.d.}}{\sim} P\), \(P \in \mathcal{P}_\varepsilon = \{P: P((X_i, Y_i) \notin V_i^*) \leq \varepsilon\}\), \(\varepsilon \in [0, 1]\)

- \(Y_i = f(X_i), f \in \mathcal{F} = \left\{f_{a,b} : \mathbb{R} \to \mathbb{R}, X \mapsto a + bX, a, b \in \mathbb{R}\right\}\)
Recapitulation: (simple) linear LIR with interval data

- \( \left( (X_i, Y_i), V_i^* \right) \overset{i.i.d.}{\sim} P, \quad P \in \mathcal{P}_\varepsilon = \{ P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon \}, \quad \varepsilon \in [0, 1] \)

- \( Y_i = f(X_i), \quad f \in \mathcal{F} = \left\{ f_{a,b} : \mathbb{R} \rightarrow \mathbb{R}, \quad X \mapsto a + bX, \quad a, b \in \mathbb{R} \right\} \)

- observations \( v_1^*, \ldots, v_n^* \) induce (normalized) profile likelihood function \( \text{lik}_{Q_{Rf}} \) of the \( p \)-quantile of the distribution of \( R_f \) for each \( f \in \mathcal{F} \)
Recapitulation: (simple) linear LIR with interval data

- \((X_i, Y_i), V_i^*\) \(\text{i.i.d.} \sim P\), \(P \in \mathcal{P}_\varepsilon = \{ P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon \} \), \(\varepsilon \in [0, 1]\)
- \(Y_i = f(X_i), f \in \mathcal{F} = \{ f_{a,b} : \mathbb{R} \rightarrow \mathbb{R}, X \mapsto a + bX, a, b \in \mathbb{R} \}\)
- observations \(v_1^*, \ldots, v_n^*\) induce (normalized) profile likelihood function \(\text{lik}_{Q_{R_f}}\) of the \(p\)-quantile of the distribution of \(R_f\) for each \(f \in \mathcal{F}\)
- \(\text{lik}_{Q_{R_f}}\) is a stepwise constant function with points of discontinuity at:
  \[0 = r_{f,(0)}, \ldots, r_{f,(\lceil n(p-\varepsilon) \rceil)}, \bar{r}_{f,(\lceil n(p+\varepsilon) \rceil + 1)}, \ldots, \bar{r}_{f,(n+1)} = +\infty\]
Recapitulation: (simple) linear LIR with interval data

- \( ((X_i, Y_i), V_i^*) \overset{i.i.d.}{\sim} P, \ P \in \mathcal{P}_\varepsilon = \{ P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon \} , \ \varepsilon \in [0, 1] \)
- \( Y_i = f(X_i) , \ f \in \mathcal{F} = \{ f_{a,b} : \mathbb{R} \to \mathbb{R} \ X \mapsto a + b X , \ a, b \in \mathbb{R} \} \)
- observations \( v_1^*, \ldots, v_n^* \) induce (normalized) profile likelihood function \( lik_{QR_f} \) of the \( p \)-quantile of the distribution of \( R_f \) for each \( f \in \mathcal{F} \)
- \( lik_{QR_f} \) is a stepwise constant function with points of discontinuity at:
  \( 0 = r_{f,(0)}, \ldots, r_{f,([n(p-\varepsilon)])}, \bar{r}_{f,([n(p+\varepsilon)]+1)}, \ldots, \bar{r}_{f,(n+1)} = +\infty \)
- \( \mathcal{C}_f = [r_{f,(k+1)}, \bar{r}_{f,\overline{k}}] , \ \text{values of } k, \overline{k} \in \mathbb{N} \cup \{0\} \ \text{depend on } n, p, \beta, \varepsilon \)
Recapitulation: (simple) linear LIR with interval data

- \((X_i, Y_i), V_i^*\) \(\overset{\text{i.i.d.}}{\sim} P,\ P \in \mathcal{P}_\varepsilon = \{P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon\}, \ \varepsilon \in [0, 1]\)
- \(Y_i = f(X_i),\ f \in \mathcal{F} = \left\{ f_{a,b} : \mathbb{R} \rightarrow \mathbb{R}, \ X \mapsto a + bX, \ a, b \in \mathbb{R} \right\}\)
- observations \(v_1^*, \ldots, v_n^*\) induce (normalized) profile likelihood function \(lik_{QR_f}\) of the \(p\)-quantile of the distribution of \(R_f\) for each \(f \in \mathcal{F}\)
- \(lik_{QR_f}\) is a stepwise constant function with points of discontinuity at:
  
\[
0 = r_{f,(0)}, \ldots, r_{f,([n(p-\varepsilon)])}, \bar{r}_{f,([n(p+\varepsilon)]+1)}, \ldots, \bar{r}_{f,(n+1)} = +\infty
\]
- \(\mathcal{C}_f = [r_{f,(k+1)}, \bar{r}_{f,(\bar{k})}],\ \text{values of } k, \bar{k} \in \mathbb{N} \cup \{0\}\) depend on \(n, p, \beta, \varepsilon\)
- LIR result \(\mathcal{U} = \{f \in \mathcal{F} : r_{f,(k+1)} \leq q_{LRM}\},\ \text{where } q_{LRM} = \inf_{f \in \mathcal{F}} \bar{r}_{f,(\bar{k})}\)
Recapitulation: (simple) linear LIR with interval data

- \( ((X_i, Y_i), V_i^*) \) \( \overset{\text{i.i.d.}}{\sim} P, \ P \in \mathcal{P}_\varepsilon = \{P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon \} \), \( \varepsilon \in [0, 1] \)

- \( Y_i = f(X_i), \ f \in \mathcal{F} = \left\{ f_{a,b} : \mathbb{R} \to \mathbb{R}, X \mapsto a + bX, \ a, b \in \mathbb{R} \right\} \)

- observations \( v_1^*, \ldots, v_n^* \) induce (normalized) profile likelihood function \( \text{lik}_{Q_Rf} \) of the \( p \)-quantile of the distribution of \( R_f \) for each \( f \in \mathcal{F} \)

- \( \text{lik}_{Q_Rf} \) is a stepwise constant function with points of discontinuity at:

- \( 0 = r_{f,(0)}, \ldots, r_{f,([n(p-\varepsilon)]), \overline{r}_f,([n(p+\varepsilon)]+1), \ldots, \overline{r}_f,(n+1) = +\infty \)

- \( \mathcal{C}_f = [r_{f,(k+1)}, \overline{r}_f,(\overline{k})] \), values of \( k, \overline{k} \in \mathbb{N} \cup \{0\} \) depend on \( n, p, \beta, \varepsilon \)

- LIR result \( \mathcal{U} = \{f \in \mathcal{F} : r_{f,(k+1)} \leq \overline{q}_{LRM}\} \), where \( \overline{q}_{LRM} = \inf_{f \in \mathcal{F}} \overline{r}_{f,(\overline{k})} \)

- if there is a unique \( f \) with \( \overline{r}_{f,(\overline{k})} = \overline{q}_{LRM} \), it is optimal according to the Likelihood-based Region Minimax (LRM) criterion and called \( f_{LRM} \)
Recapitulation: (simple) linear LIR with interval data

- \((X_i, Y_i), V_i^*) \overset{i.i.d.}{\sim} P, \ P \in \mathcal{P}_\varepsilon = \{P : P((X_i, Y_i) \notin V_i^*) \leq \varepsilon\}, \ \varepsilon \in [0, 1]

- \(Y_i = f(X_i), \ f \in \mathcal{F} = \left\{ f_{a,b} : \mathbb{R} \to \mathbb{R}, \ X \mapsto a + bX, \ a, b \in \mathbb{R} \right\}\)

- Observations \(v_1^*, \ldots, v_n^*\) induce (normalized) profile likelihood function \(\text{lik}_{Q_{R_f}}\) of the \(p\)-quantile of the distribution of \(R_f\) for each \(f \in \mathcal{F}\)

- \(\text{lik}_{Q_{R_f}}\) is a stepwise constant function with points of discontinuity at:
  \[0 = r_{f,(0)}, \ldots, r_{f,\lfloor n(p-\varepsilon) \rfloor}, \bar{r}_{f,\lfloor n(p+\varepsilon) \rfloor+1}, \ldots, \bar{r}_{f,(n+1)} = +\infty\]

- \(C_f = [r_{f,(k+1)}, \bar{r}_{f,\bar{k}}]\), values of \(k, \bar{k} \in \mathbb{N} \cup \{0\}\) depend on \(n, p, \beta, \varepsilon\)

- LIR result \(\mathcal{U} = \left\{ f \in \mathcal{F} : r_{f,(k+1)} \leq \bar{q}_{LRM} \right\}\), where \(\bar{q}_{LRM} = \inf_{f \in \mathcal{F}} \bar{r}_{f,(\bar{k})}\)

- If there is a unique \(f\) with \(\bar{r}_{f,(\bar{k})} = \bar{q}_{LRM}\), it is optimal according to the Likelihood-based Region Minimax (LRM) criterion and called \(f_{LRM}\)

Implementation: Exact algorithm for simple linear LIR

\[ U = \{ f \in F : r_f, (k+1) \leq q_{LRM} \} \]

- 1st step: find \( q_{LRM} \)
- \( \beta = 0 \), \( p = 0 \), \( n = 17 \), and \( k = 12 \)
Implementation: Exact algorithm for simple linear LIR

- aim: determine the set of all undominated functions $\mathcal{U} = \{ f \in \mathcal{F} :\]

\[ r_f,(k+1) \leq \bar{q}_{LRM} \}$
Implementation: Exact algorithm for simple linear LIR

- **aim:** determine the set of all undominated functions $\mathcal{U} = \{ f \in \mathcal{F} : r_{f,(k+1)} \leq \overline{q}_{LRM} \}$
- **1st step:** find $\overline{q}_{LRM}$
Implementation: Exact algorithm for simple linear LIR

- aim: determine the set of all undominated functions \( \mathcal{U} = \{ f \in \mathcal{F} : \overline{r}_{f,(k+1)} \leq \overline{q}_{LRM} \} \)

- 1st step: find \( \overline{q}_{LRM} \)

- \( B_{f_{LRM},\overline{q}_{LRM}} \) (blue dashed lines) is the thinnest band containing at least \( k \) imprecise data

- here \( \beta = 0.8, p = 0.6, n = 17 \), and \( k = 12 \)
some of the included \( \bar{k} \) imprecise data touch the border of \( \overline{B}_{f_{LRM},q_{LRM}} \) in 3 different points
some of the included \( \bar{k} \) imprecise data touch the border of \( \overline{B}_{f_{L_{RM}}, \overline{q}_{L_{RM}}} \) in 3 different points

\( b_{L_{RM}} \) can be any slope determined by the corresponding corner points of 2 imprecise data or 0
some of the included $\bar{k}$ imprecise data touch the border of $\overline{B}_{f_{LRM}, \overline{q}_{LRM}}$ in 3 different points

$b_{LRM}$ can be any slope determined by the corresponding corner points of 2 imprecise data or 0

$\mathcal{B}$: set of all $4 \binom{n}{2} + 1$ possible values for $b_{LRM}$
• some of the included $\bar{k}$ imprecise data touch the border of $B_{f_{LRM},\bar{q}_{LRM}}$ in 3 different points

• $b_{LRM}$ can be any slope determined by the corresponding corner points of 2 imprecise data or 0

• $\mathcal{B}$: set of all $4 \binom{n}{2} + 1$ possible values for $b_{LRM}$

• for each $b \in \mathcal{B}$ find $a_b \in \mathbb{R}$ for which $\bar{r}_{f_{ab},b,\bar{k}}$ is minimal
Implementation: Exact algorithm - Part 1

for each $b \in B$

- consider transformed data $z^*_b, i = [z_{b,i}, \bar{z}_{b,i}]$ with

$$z_{b,i} = \begin{cases} y_i - b \bar{x}_i, & b > 0 \\ y_i - b \underline{x}_i, & b \leq 0 \end{cases} \quad \text{and} \quad \bar{z}_{b,i} = \begin{cases} \bar{y}_i - b \bar{x}_i, & b > 0 \\ \bar{y}_i - b \underline{x}_i, & b \leq 0 \end{cases}$$
Implementation: Exact algorithm - Part 1

for each \( b \in B \)

- consider transformed data \( z_{b,i}^* = [\underline{z}_{b,i}, \overline{z}_{b,i}] \) with

\[
\underline{z}_{b,i} = \begin{cases} 
  \bar{y}_i - b \bar{x}_i, & b > 0 \\
  \bar{y}_i - b \bar{x}_i, & b \leq 0 
\end{cases}
\]

and

\[
\overline{z}_{b,i} = \begin{cases} 
  \bar{y}_i - b \bar{x}_i, & b > 0 \\
  \bar{y}_i - b \bar{x}_i, & b \leq 0 
\end{cases}
\]

- for example, \( z_{b,i}^* \) for \( b = -0.25 \), ordered by lower endpoint

\[
\begin{array}{c|c|c}
\text{(i)} & \text{Z}_{b,(i)} & \text{Z}_{b,(i)} \\
\hline
0 & 0 & \text{Z}_{b,(i)} \\
1 & 5 & \text{Z}_{b,(i)} \\
2 & 10 & \text{Z}_{b,(i)} \\
3 & 15 & \text{Z}_{b,(i)} \\
\end{array}
\]
for each $b \in B$

- consider transformed data $z_{b,i}^* = [z_{b,i}, \bar{z}_{b,i}]$ with

$$z_{b,i} = \begin{cases} y_i - b \bar{x}_i, & b > 0 \\ \bar{y}_i - b \bar{x}_i, & b \leq 0 \end{cases}$$

and

$$\bar{z}_{b,i} = \begin{cases} \bar{y}_i - b \bar{x}_i, & b > 0 \\ y_i - b \bar{x}_i, & b \leq 0 \end{cases}$$

- for example, $z_{b,i}^*$ for $b = -0.25$, ordered by lower endpoint

- find the shortest interval $I_b$ containing at least $\bar{k}$ of the transformed data $z_{b,i}^*$, i.e., determine the shortest of the $n - \bar{k} + 1$ intervals of the form $[z_{b,(j)}, \bar{z}_{b,[j]}]$, where $\bar{z}_{b,[j]}$ is the $\bar{k}$th smallest value among the $\bar{z}_{b,i}$ such that $z_{b,i} \geq \bar{z}_{b,(j)}$
for each $b \in \mathcal{B}$

- the length of $\mathcal{I}_b$ corresponds to the width of the closed band around the function $f_{a_b,b}$ containing at least $k$ imprecise data
for each \( b \in B \)

- the length of \( I_b \) corresponds to the width of the closed band around the function \( f_{a_b,b} \) containing at least \( k \) imprecise data
- the corresponding intercept \( a_b \) is given by the midpoint of the interval \( I_b \)
- for example, \( b = -0.25 \)
Implementation: Exact algorithm - Part 1

for each $b \in B$

- the length of $\mathcal{I}_b$ corresponds to the width of the closed band around the function $f_{a_b,b}$ containing at least $\bar{k}$ imprecise data
- the corresponding intercept $a_b$ is given by the midpoint of the interval $\mathcal{I}_b$
- for example, $b = -0.25$

- the associated $\bar{r}_{f_{a_b,b},(\bar{k})}$ corresponds to half of the length of $\mathcal{I}_b$
Implementation: Exact algorithm - Part 1

for each \( b \in B \)

- the length of \( \mathcal{I}_b \) corresponds to the width of the closed band around the function \( f_{ab}, b \) containing at least \( \bar{k} \) imprecise data
- the corresponding intercept \( a_b \) is given by the midpoint of the interval \( \mathcal{I}_b \)
- for example, \( b = -0.25 \)

\[
\bar{q}_{LRM} = \frac{1}{2} \min_{(b,j) \in B \times \{1, \ldots, n-k+1\}} (\bar{Z}_b[j] - Z_{b,(j)})
\]
step 2: determine $U$
step 2: determine $U$

for $f \in U$, the band $\overline{B_{f,q_{LRM}}}$ intersects at least $k + 1$ data

here $k = 8$
step 2: determine $\mathcal{U}$

- for $f \in \mathcal{U}$, the band $B_f, q_{LRM}$ intersects at least $k + 1$ data
- here $k = 8$

- for $b \in \mathbb{R}$ find all intercept values $a \in \mathbb{R}$, for which $r_{f_a, b, (k+1)} \leq q_{LRM}$
Implementation: Exact algorithm - Part 2

for a given $b \in \mathbb{R}$

- consider again the transformed data $z_{b,i}^*$
Implementation: Exact algorithm - Part 2

for a given \( b \in \mathbb{R} \)

- consider again the transformed data \( z_{b,i}^* \)
- \( A_{b,l} \) is the set of interval midpoints \( a \in \mathbb{R} \), for which the interval \( [a - q_{LRM}, a + q_{LRM}] \) intersects all \( z_{b,i}^* \) of the \( l \)th subset of size \( k + 1 \)
Implementation: Exact algorithm - Part 2

for a given \( b \in \mathbb{R} \)

- consider again the transformed data \( z_{b,i}^* \)
- \( A_{b,l} \) is the set of interval midpoints \( a \in \mathbb{R} \), for which the interval \([a - \bar{q}_{LRM}, a + \bar{q}_{LRM}]\) intersects all \( z_{b,i}^* \) of the \( l \)th subset of size \( k + 1 \)
- for example, \( b = -0.25 \)
for a given $b \in \mathbb{R}$

- consider again the transformed data $z_{b,i}^{*}$
- $A_{b,l}$ is the set of interval midpoints $a \in \mathbb{R}$, for which the interval $[a - \bar{q}_{LRM}, a + \bar{q}_{LRM}]$ intersects all $z_{b,i}^{*}$ of the $l$th subset of size $k + 1$
- for example, $b = -0.25$

$$
\mathcal{A}_b = \bigcup_{j=1}^{n-k} [z_{b,(k+j)} - \bar{q}_{LRM}, z_{b,j} + \bar{q}_{LRM}]
$$
for a given $b \in \mathbb{R}$

- consider again the transformed data $z_{b,i}^*$

- $A_{b,l}$ is the set of interval midpoints $a \in \mathbb{R}$, for which the interval $[a - \bar{q}_{LRM}, a + \bar{q}_{LRM}]$ intersects all $z_{b,i}^*$ of the $l$th subset of size $k + 1$

- for example, $b = -0.25$

the union of all $A_{b,l}$ is equivalent to $A_b = \bigcup_{j=1}^{n-k} [z_{b,(k+j)} - \bar{q}_{LRM}, \bar{z}_{b,(j)} + \bar{q}_{LRM}]$

finally, we obtain $\mathcal{U}$ as the set $\{f_{a,b} : b \in \mathbb{R} \text{ and } a \in A_b\}$
Resulting set of undominated parameters

A. Wiencierz and M. Cattaneo (LMU Munich) Exact algorithm for linear LIR with interval data  SMPS 6, Oct. 4, 2012 13 / 19
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $U$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$

linLIR package
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $U$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$

linLIR package

- linLIR: linear Likelihood-based Imprecise Regression, available at CRAN: http://cran.r-project.org/
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$

linLIR package

- linLIR: linear Likelihood-based Imprecise Regression, available at CRAN: http://cran.r-project.org/
- function to plot 2-dimensional interval data set, two example data sets
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity \( O(n^3 \log n) \)

linLIR package

- linLIR: linear Likelihood-based Imprecise Regression, available at CRAN: http://cran.r-project.org/
- function to plot 2-dimensional interval data set, two example data sets
- s.linlir function implements the exact algorithm
Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $U$, i.e., the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$

linLIR package

- function to plot 2-dimensional interval data set, two example data sets
- `s.linlir` function implements the exact algorithm
- further tools to summarize and visualize results
Example - Data set

- 2-dimensional interval data set of $n = 514$ observations
Example - Data set

- 2-dimensional interval data set of $n = 514$ observations
- LIR analysis with $\rho = 0.5, \beta = 0.26, \varepsilon = 0$
Example - Data set

- 2-dimensional interval data set of $n = 514$ observations
- LIR analysis with $p = 0.5$, $\beta = 0.26$, $\varepsilon = 0$
- $\underline{k} = 238$, $\overline{k} = 276$
Example - R code

```r
library(linLIR)
data(pm10)
pm.idf <- idf.create(pm10, var.labels=c("X","Y"))
pm.lir <- s.linlir(pm.idf, p = 0.5, bet=0.26, epsilon = 0)
summary(pm.lir)

Ranges of parameter values of the undominated functions:
intercept of f in [8.977766,27.18173]
slope of f in [0.11,1.898]

Bandwidth:  10.79207

Estimated parameters of the function f.lrm:
intercept of f.lrm:  18.39075
slope of f.lrm:  1.059185

Number of observations:  514

LIR settings:
p:  0.5 beta:  0.26 epsilon:  0 k.l:  238 k.u:  276
confidence level of each confidence interval:  90.61 %
```
Example - Undominated regression functions

- 2-dimensional interval data set of $n = 514$ observations
- LIR analysis with $p = 0.5, \beta = 0.26, \varepsilon = 0$
- $k = 238$, $\bar{k} = 276$
- obtained set of all undominated regression functions
Example - Undominated parameters

- 2-dimensional interval data set of $n = 514$ observations
- LIR analysis with $p = 0.5, \beta = 0.26, \varepsilon = 0$
- $k = 238, \overline{k} = 276$
- obtained set of all undominated regression functions
- obtained set of parameters
Summary and Outlook
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations.
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of \( X \) and \( Y \) in the light of the imprecise observations
- for the special case of simple linear regression with interval data, we developed an exact algorithm to determine the set of all undominated functions
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- for the special case of simple linear regression with interval data, we developed an exact algorithm to determine the set of all undominated functions
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- for the special case of simple linear regression with interval data, we developed an exact algorithm to determine the set of all undominated functions
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations for the special case of simple linear regression with interval data, we developed an exact algorithm to determine the set of all undominated functions.
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression.
- the presented algorithm has computational complexity $O(n^3 \log n)$.
- the exact algorithm is implemented in R as part of the linLIR package.
Summary and Outlook

- the imprecise result of the LIR analysis is the set of all functions that are plausible relations of $X$ and $Y$ in the light of the imprecise observations
- for the special case of simple linear regression with interval data, we developed an exact algorithm to determine the set of all undominated functions
- the first part of the algorithm generalizes the exact algorithm for Least Quantile of Squares regression
- the presented algorithm has computational complexity $O(n^3 \log n)$
- the exact algorithm is implemented in R as part of the linLIR package
- future work: generalize algorithm to multiple linear regression